Here we derive Integration by Parts from the Product Rule for derivatives. We will abbreviate $f(x)$, $g(x)$, $f'(x)$ and $g'(x)$ as f, g, f', and g' respectively.

$$\begin{aligned}
(fg)' &= f'g + fg' \\
\int (fg)'dx &= \int (f'g + fg')dx \\
fg &= \int f'gdx + \int fg'dx \\
fg - \int f'gdx &= \int fg'dx
\end{aligned}$$

so when given an integral that can be written as $\int fg'dx$, we can rewrite it as $fg - \int f'gdx$. We can thus fill in the 2-by-2 grid:

$$\begin{array}{ll}
f = & f' = \\
g = & g' =
\end{array}$$

For example, given $\int x \sin x dx$, we can write

$$\begin{array}{ll}
f = x & f' = \\
g = & g' = \sin x
\end{array}$$

and fill in the other parts (taking a derivative of f and an antiderivative of g'):

$$\begin{array}{ll}
f = x & f' = 1 \\
g = -\cos x & g' = \sin x
\end{array}$$

And the Integration by Parts rule gives

$$\begin{aligned}
\int x \sin x dx &= \int fg'dx \\
&= fg - \int f'gdx \\
&= x(-\cos x) - \int 1(-\cos x)dx \\
&= -x\cos x - \int -\cos x dx \\
&= -x\cos x + \int \cos x dx \\
&= -x\cos x + \sin x + C
\end{aligned}$$